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14. ABSTRACT

The purpose of the proposed research project was to address the problem of communication black-out with a vehicle during a sustained hypersonic flight. Attempts to communicate through dense plasmas, which forms around an aircraft during hypersonic flight, fail when the transmitted wave frequency is below the plasma frequency. Under such conditions, the transmitted propagating wave becomes evanescent inside the plasma layer and its amplitude is exponentially attenuated. It was demonstrated that without accounting for dissipation, anomalous plasma transparency to radio-frequencies can be achieved via amplification of evanescent waves using a diffraction grating, periodic plasma, or a barrier. For dissipative plasma, adequate transmission can be achieved using a diffraction grating or periodic modulation. Furthermore, using a barrier, reflectionless transmission and absorption can be achieved in dissipative plasma.

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FINAL TECHNICAL REPORT

for May 1, 2007—November 30, 2009

Transmission of radio-frequency signals through plasma during hypersonic flight

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Research proposal: #FA9550-07-1-0415, The United States Air Force, Air Force Office of Scientific Research

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Abstract

The purpose of the proposed research project was to address the problem of communication black-out with a vehicle during a sustained hypersonic flight. Attempts to communicate through dense plasmas, which forms around an aircraft during hypersonic flight, fail when the transmitted wave frequency is below the plasma frequency. Under such conditions, the transmitted propagating wave becomes evanescent inside the plasma layer and its amplitude is exponentially attenuated. It was demonstrated that without accounting for dissipation, anomalous plasma transparency to radio-frequencies can be achieved via amplification of evanescent waves using a diffraction grating, periodic plasma modulation, or a barrier. For dissipative plasma, adequate transmission can be achieved using a diffraction grating or periodic modulation. Furthermore, using a barrier, reflectionless transmission and absorption can be achieved in dissipative plasma.

Status of Effort

The purpose of the proposed research project was to address the problem of communication black-out with a vehicle during a sustained hypersonic flight. Attempts to communicate through dense plasmas, which forms around an aircraft during hypersonic flight, fail when the transmitted wave frequency is below the plasma frequency. Under such conditions, the transmitted propagating wave becomes evanescent inside the plasma layer and its amplitude is exponentially attenuated. The main goal was to develop a theoretical framework for achieving anomalous plasma transparency to radio-frequencies.

The main idea of the proposed method was to amplify the evanescent wave through resonance with the surface wave, and then use that evanescent wave to transmit a GPS signal through dense plasma. In order to achieve resonance, it was proposed to use periodic plasma modulation, induced by a diffraction grating. Similar methods have recently been used in near field optics and nano-photonics. It seemed natural to ask whether such an approach can be used to solve the communications problem.

During the first period of the research project, we concentrated on formulating mathematical models for the communication problem which take relevant plasma properties under consideration. We worked in three different, but related directions:

- formulating and studying the transmission problem for homogeneous plasmas;

- understanding the properties and geometry of the non-homogeneous bounded plasma that forms during a hypersonic flight;

- formulating and studying the transmission problem for non-homogeneous plasma.

In particular, we have demonstrated that plasma transparency can be achieved through periodic plasma modulation for plasma parameters which occur during a hypersonic flight.

During the second year of the research project, we investigated various resonance conditions which could be relevant for hypersonic flights and further our understanding of signal transmission through dense plasma. In particular, we concentrated on specific problems related to electromagnetic wave transmission, such as

- resonant transmission in multi-layer dense plasma structures;

- standing wave resonance in a multi-layer structure;

- resonant transmission through dense plasma and a partially modulated boundary layer;

• resonant modes and tunneling in multi-layer structures;

• effects of electron-atom collisions;

• modeling two-dimensional bounded plasmas for intermediate and high pressures.

In particular, we have made the first attempt to achieve resonant transmission using a simplified model of a diffraction grating.

In the third and final period of the research project, we have concentrated on signal transmission using a diffraction grating. In particular, we formulated a general mathematical model which employs a diffraction grating. During this period, we have been working in the following directions:

• developing a model which describes signal transmission through dense plasma using a diffraction grating;

• understanding the parameters of the diffraction grating which are needed to achieve plasma transparency;

• understanding plasma transparency for different incidence angles;

• understanding resonance induced by periodic plasma modulation;

• understanding resonance induced by a barrier;

• effects of dissipation on signal transmission;

• modeling two-dimensional bounded collisional plasma;

• understanding effects of oblique magnetic fields.

The research has been done in collaboration with Andrei Smolyakov, Department of Physics, University of Saskatchewan, Canada and Valery Godyak, RF Plasma Consulting, Brookline, MA.

Publications

1. N. Sternberg and V. Godyak, The Bohm plasma-sheath model and the Bohm criterion revisited, Special Issue, IEEE Trans. Plasma Sci., 35, 5, 1341, 2007.

2. V. Godyak and N. Sternberg, Two-dimensional cylindrical plasma at low gas pressure, Plasma Sources Sci. and Techn., 17, 025004, 2008.

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8. N. Sternberg and A. I. Smolyakov, Resonant transmission through dense plasmas via amplification of evanescent mode, PIERS ONLINE, vol. 5, no. 8, 2009, p. 781.

9. N. Sternberg and A. Smolyakov, Resonant transparency of a three-layer structure containing the dense plasma region, Progress In Electromagnetics Research, PIER 99, 37–52, 2009.

10. A. Smolyakov, S. I. Krashennikov, E. Furkal and N. Sternberg, Resonant modes and tunneling in multi-layer metal dielectric structures (to be submitted).

Work in Progress

1. N. Sternberg and A. Smolyakov, Resonant transmission of electromagnetic waves using a diffraction grating.

2. A. Smolyakov and N. Sternberg, Resonance transmission of electromagnetic waves induced by plasma modulation.

3. N. Sternberg and A. Smolyakov, Reflectionless transmission and absorption in dense plasma with a barrier.

4. N. Sternberg and A. Smolyakov, Effects of dissipation of electromagnetic wave transmission through dense plasma.

5. N. Sternberg and V. Godyak, Two-dimensional collisional discharge plasma.

6. N. Sternberg and J.-L. Raimbault, Effects of oblique magnetic fields on plasmas.

Conference Presentations

1. "Anomalous transparency of opaque materials", Canadian Association of Physicists Congress, June 18-20, 2007 Saskatoon, Canada.

2. "The Bohm plasma-sheath model and the Bohm criterion revisited", 60 Gaseous Electronics Conference, Washington, DC, October 2–5, 2007.

3. "Two-dimensional collisionless weakly ionized plasma in fluid approximation", 60 Gaseous Electronics Conference, Washington, DC, October 2–5, 2007

4. "Influence of Ramsauer effect on bounded plasmas in magnetic fields", 35 European Plasma Conference, June 13–June 19, 2008, Crete, Greece.

5. "Two-dimensional gas discharge plasma", 2009 Workshop on Radio-Frequency Discharges, La Presqu'il de Giens, France, 17–20 May, 2009.

6. "Resonant transparency of multi-layer structures with opaque materials", 9th International Conference on Mathematical and Numerical Aspects of Wave Propagation, 15-19 June, 2009, Pau, France.

7. "Resonant transmission through dense plasmas via amplification of evanescent mode", Progress in Electromagnetics Research Symposium (PIERS) 2009, Moscow, Russia, 18-21 August, 2009.

Presentations at the AFOSR T&E Portfolio Review

1. "Communication through plasma", Washington, DC, September 18–19, 2007.

2. "Communication through plasma during hypersonic flight", Reston, VI, August 5-6, 2008.

3. "Communication through plasma during hypersonic flight", Dayton, OH, September 21–23, 2009.

Main Results

1. Some theoretical aspects of resonant transmission induced by a diffraction grating.

Resonant transmission induced by a diffraction grating was first studied in [1], [2]. There, dense plasma was bounded by a diffraction grating on each side. Such symmetric configuration, however, is not applicable for conditions that occur during hypersonic flight, where one could expect to use only one diffraction grating at the antenna.

In our model, we considered the configuration shown in Fig. 1. A dense plasma layer of width $l = b - a$ is nested between a semi-infinite vacuum layer and a dielectric layer of positive permittivity. A diffraction grating is placed inside the dielectric layer at a distance a from the plasma layer. We consider the regions to be in the (y, z)-plane, and we set $z = 0$ at the position of the diffraction grating. In this way, we obtain four distinct regions: the semi-infinite vacuum region V to the left of the plasma layer $(z < -b)$, the plasma layer P $(-b < z < -a)$, the dielectric region D_1 $(-a < z < 0)$ to the left of the diffraction grating, and the dielectric layer D_2 $(z > 0)$ of an arbitrary width to the right of the diffraction grating. For simplicity of exposition, we assume that the dielectric regions D_1 and D_2 have the same permittivity.

In a general medium, the dispersion relation of the incidence wave of frequency ω

$$H = H_0 \exp(ik_z z + ik_y y - i\omega t) \tag{1}$$

is given by

$$\frac{\omega^2}{c^2}\epsilon = k_z^2 + k_y^2 \tag{2}$$

where c is the speed of light. For vacuum, $\epsilon = 1$, for dense plasma $\epsilon = \epsilon_p < 0$, and for the dielectric material $\epsilon = \epsilon_d > 1$. Since $k_y = (\omega/c)\sin(\theta) < \omega/c$, as long as $\epsilon \geq 1$, it holds that $k_z^2 > 0$, and the wave is propagating. In the plasma, however, $k_z^2 < 0$, the wave is evanescent and decays rapidly. This is illustrated in Fig. 2.

Although the wave in the plasma region is decaying, two modes exist in that region, one exponentially decaying and the other one exponentially growing. It was shown in [3] that if both exponential modes are combined they carry energy and can be used for signal transmission. The question becomes how to increase the amplitude of the transmitted incident wave inside the dense plasma layer. It is well known that this can be achieved using surface wave resonance. Surface waves form at a sharp interfaces between a medium with positive permittivity and a medium with negative permittivity. The dispersion relation of the surface wave between two media

of permittivity ϵ_1 and ϵ_2 is given by

$$\frac{\gamma(\epsilon_1)}{\epsilon_1} = -\frac{\gamma(\epsilon_2)}{\epsilon_2} \tag{3}$$

where

$$\gamma^2 = k_y^2 - \epsilon\frac{\omega^2}{c^2} \tag{4}$$

which yields the dispersion relation

$$k_y^2 = \frac{\omega^2}{c^2}\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2} \tag{5}$$

If a wave is in resonance with the surface wave, then the wave vector k_y for that wave has to satisfy (5). Obviously, it is not possible to achieve surface wave resonance at the plasma-vacuum or plasma-dielectric interfaces, and the incident signal cannot be amplified as it passes through those different regions.

By placing a diffraction grating inside the dielectric region, two sidebands are generated:

$$H^+ = H_+(z)\exp(i(k_y+q)y - i\omega t) \tag{6}$$

and

$$H^- = H_-(z)\exp(i(k_y-q)y - i\omega t)) \tag{7}$$

If at least one of the sidebands is in resonance with the surface wave, one of the following dispersion relations is satisfied:

$$(k_y+q)^2 = \frac{\omega^2}{c^2}\frac{\epsilon_p\epsilon_d}{\epsilon_p+\epsilon_d} \tag{8}$$

or

$$(k_y-q)^2 = \frac{\omega^2}{c^2}\frac{\epsilon_p\epsilon_d}{\epsilon_p+\epsilon_d} \tag{9}$$

This yields the value of the diffraction grating wave vector, q, which is needed to achieve resonance between the sidebands and the surface wave.

The sidebands and the incidence wave are independent. Each satisfies the wave equation. Passing through the region, each wave satisfies continuity conditions at the corresponding interface. At the diffraction grating, however, all waves are coupled through the corresponding boundary condition. If the amplitude of the sidebands becomes large due to surface wave resonance, then the amplitude of the incident wave will be amplified. Thus, the incidence wave is not amplified directly by the surface wave. The surface wave amplifies the sidebands through resonance, and the sidebands amplify the incidence wave.

We have found that the theory developed above can be applied to plasmas under hypersonic flight conditions. This is illustrated in Fig. 3 when the plasma dissipation is neglected. For the plasma parameters, we have chosen the values suggested in [4] and [5]: the plasma width is $l = 0.02$m and the plasma frequency is $\omega_{pe}/(2\pi) = 6$GHz. We chose the incidence frequency within the GPS range: $\omega/(2\pi) = 1$GHz. Then, neglecting dissipation, the plasma permittivity is $\epsilon_p = -35$.

Our preliminary results were presented at two conferences:

(i) 9th International Conference on Mathematical and Numerical Aspects of Wave Propagation, 15-19 June, 2009, Pau, France.

(ii) Progress in Electromagnetics Research Symposium (PIERS) 2009, Moscow, Russia, 18-21 August, 2009.

Some of our preliminary results can be found in the following publications:

(i) N. Sternberg and A. Smolyakov, Resonant transmission through dense plasmas via amplification of evanescent mode, PIERS Proceedings, Moscow, Russia, August 18–21, 2009, p.1618.

(ii) N. Sternberg and A. I. Smolyakov, Resonant transmission through dense plasmas via amplification of evanescent mode, PIERS ONLINE, vol. 5, no. 8, 2009, p. 781.

2. Modeling wave transmission through dense plasma with a diffraction grating.

As in the previous section, we consider the configuration shown in Fig. 1. Suppose an electromagnetic incident wave propagates in the vacuum region V toward the plasma layer with the incidence angle θ. The question is whether in the given configuration, a modulation by the diffraction grating can result in resonant transmission of the incident wave through the dense plasma layer if the plasma frequency ω_{pe} is above the wave frequency ω. To answer this question, we will use the Fourier approximation only up to the first harmonics.

We consider the p-polarization: $H = (H_x, 0, 0)$, and $E = (0, E_y, E_z)$. In each region, the incident wave is periodic in the y-direction, but becomes evanescent in the z-direction when it enters the dense plasma layer. Thus, we can represent the incident magnetic wave in the form $H_0(z)e^{ik_y y - i\omega t}$ with $k_y = (\omega/c)\sin(\theta)$. As a result of the modulation by the diffraction grating, two sidebands are induced, both periodic in the y-direction, one with the wave vector $k_y + q$, the other with the wave vector $k_y - q$, where q is the wave vector of the diffraction grating. In order to achieve resonant amplification of the amplitude of the evanescent incident wave, the

sidebands have to be evanescent in the z-direction. We therefore represent them in the form $H_+(z)e^{i(k_y+q)y-i\omega t}$ and $H_-(z)e^{i(k_y-q)y-i\omega t}$. Altogether, we are looking for the magnetic wave field of the form

$$H_x(y,z,t) = [H_0(z)e^{ik_y y} + H_+(z)e^{i(k_y+q)y} + H_-(z)e^{i(k_y-q)y}]e^{-i\omega t} \tag{10}$$

as it is transmitted through all regions.

Recall that for a double non-homogenous medium (in y and in z) the Maxwell equations yield the following wave equation:

$$\varepsilon\frac{\partial}{\partial y}\left(\frac{1}{\varepsilon}\frac{\partial H_x}{\partial y}\right) + \varepsilon\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\frac{\partial H_x}{\partial z}\right) + \frac{\omega^2}{c^2}\varepsilon H_x = 0 \tag{11}$$

where ε is the permittivity. We assume that ε is defined by the following step-function: $\varepsilon(y,z) = 1$ if (y,z) is in the vacuum region, $\varepsilon(y,z) = \varepsilon_d$ if (y,z) is in the dielectric region, and $\varepsilon(y,z) = \varepsilon_p = 1 - \omega_{pe}^2/(\omega^2 + i\nu_e\omega)$ if (y,z) is in the plasma region. Here, ν_e is the electron-atom collision frequency. Thus, each of the regions is homogeneous, and there is a sharp interface between them. Note that $\varepsilon_p < 0$ for a dense plasma where $\omega_{pe} > \omega$ and $\nu_e = 0$.

In order to account for the diffraction grating, we modify (11) as follows:

$$\varepsilon\frac{\partial}{\partial y}\left(\frac{1}{\varepsilon}\frac{\partial H_x}{\partial y}\right) + \varepsilon\frac{\partial}{\partial z}\left(\frac{1}{\varepsilon}\frac{\partial H_x}{\partial z}\right) + \frac{\omega^2}{c^2}\varepsilon H_x + \frac{\omega^2}{c^2}h_g\left(\varepsilon_g + \alpha\cos\left(qy\right)\right)\delta\left(z\right)H_x = 0 \tag{12}$$

where α is the modulation parameter, ε_g is the permittivity of the diffraction grating and h_g is its width, δ is the δ-function. Note that outside the diffraction grating, equations (11) and (12) coincide. Equation (12) is similar to the one found in [1], [2], but it is somewhat more general.

At the plasma-vacuum interface and at the plasma-dielectric interface, the Maxwell equations yield continuity of H_x and of $\varepsilon^{-1}dH_x/dz$, i.e., at $z = -b$ and $z = -a$,

$$[H_x]_-^+ = 0 \qquad \text{and} \qquad \left[\frac{1}{\varepsilon}\frac{dH_x}{dz}\right]_-^+ = 0 \tag{13}$$

where "+" and "−" indicate the corresponding limits in the z direction from the right and from the left of the interface for all y. At the diffraction grating, H_x is continuous, and integrating equation (12) yields

$$\left[\frac{dH_x}{dz}\right]_-^+ = -\Big(k_g + \frac{k_\alpha}{2}(e^{iqy} + e^{-iqy})\Big)H_x(0) \tag{14}$$

where

$$k_g = \frac{\omega^2}{c^2}\epsilon_g h_g \qquad \text{and} \qquad k_\alpha = \frac{\omega^2}{c^2}h_g\alpha \tag{15}$$

In order to find the corresponding wave equations for the incident wave and the sidebands in each region, we substitute (10) into (11) and obtain:

$$\frac{d^2H_0}{dz^2} - k_y^2 H_0 + \frac{\omega^2}{c^2}\epsilon H_0 = 0 \tag{16}$$

$$\frac{d^2H_+}{dz^2} - (k_y+q)^2 H_+ + \frac{\omega^2}{c^2}\epsilon H_+ = 0 \tag{17}$$

$$\frac{d^2H_-}{dz^2} - (k_y-q)^2 H_- + \frac{\omega^2}{c^2}\epsilon H_- = 0 \tag{18}$$

The boundary conditions (13) and (14) yield the following boundary conditions for the incident wave and the sidebands:

$$[H_0]_-^+ = 0 \qquad [H_+]_-^+ = 0 \qquad [H_-]_-^+ = 0 \tag{19}$$

at the plasma-vacuum interface, the plasma-dielectric interface and at the diffraction grating. Furthermore,

$$\Big[\frac{1}{\varepsilon}\frac{dH_0}{dz}\Big]_-^+ = 0 \qquad \Big[\frac{1}{\varepsilon}\frac{dH_+}{dz}\Big]_-^+ = 0 \qquad \Big[\frac{1}{\varepsilon}\frac{dH_-}{dz}\Big]_-^+ = 0 \tag{20}$$

at the plasma-vacuum interface and the plasma-dielectric interface. Moreover,

$$\Big[\frac{dH_0}{dz}\Big]_-^+ = -k_g H_0(0) - \frac{k_\alpha}{2}H_+(0) - \frac{k_\alpha}{2}H_-(0) \tag{21}$$

$$\Big[\frac{dH_+}{dz}\Big]_-^+ = -k_g H_+(0) - \frac{k_\alpha}{2}H_0(0) \tag{22}$$

$$\Big[\frac{dH_-}{dz}\Big]_-^+ = -k_g H_-(0) - \frac{k_\alpha}{2}H_0(0) \tag{23}$$

at the diffraction grating. Here, as above, "+" and "−" indicate the corresponding limit from the right and from the left along the z axis. Note that the incident wave and the sidebands are coupled through the boundary conditions at the diffraction grating.

In each region shown in Fig. 1, the general solution of the equations (16), (17) and (18) is of the form

$$H_0 = A(\exp(\gamma z) + \Gamma \exp(-\gamma z)) \tag{24}$$

$$H_+ = A^+(\exp(\gamma^+ z) + \Gamma^+ \exp(-\gamma^+ z)) \tag{25}$$

$$H_- = A^-(\exp(\gamma^- z) + \Gamma^- \exp(-\gamma^- z)) \tag{26}$$

where

$$\gamma^2 = k_y^2 - \frac{\omega^2}{c^2}\varepsilon \tag{27}$$

$$(\gamma^+)^2 = (k_y + q)^2 - \frac{\omega^2}{c^2}\varepsilon \tag{28}$$

$$(\gamma^-)^2 = (k_y - q)^2 - \frac{\omega^2}{c^2}\varepsilon \tag{29}$$

Note that A, A^+ and A^- denote the amplitudes of the corresponding incident waves, while Γ, Γ^+ and Γ^- are the corresponding reflection coefficients.

The behavior of H_0, H_+ and H_- depends on each region. In the vacuum region, $\varepsilon = 1$, $k_y < \omega/c$, and the wave H_0 is propagating. Hence,

$$\gamma = i\gamma_v \tag{30}$$

where

$$\gamma_v = \sqrt{\frac{\omega^2}{c^2} - k_y^2} = \frac{\omega}{c}\cos(\theta) \tag{31}$$

In the plasma region, H_0 is evanescent, and

$$\gamma = -\gamma_p \tag{32}$$

where

$$\gamma_p = \sqrt{k_y^2 - \frac{\omega^2}{c^2}\epsilon_p} = \frac{\omega}{c}\sqrt{\sin^2(\theta) - \epsilon_p} \tag{33}$$

The transmitted wave is propagating in the dielectric region. Thus, $\varepsilon_d > \sin^2(\theta)$, and

$$\gamma = i\gamma_d \tag{34}$$

where

$$\gamma_d = \frac{\omega}{c}\sqrt{\varepsilon_d - \sin^2(\theta)} \tag{35}$$

As mentioned above, in order to induce resonance with the evanescent incident wave, the sidebands H_+ and H_- have to be evanescent. This is achieved if $q > k_y + (\omega/c)\sqrt{\varepsilon_d}$. Furthermore, the sidebands are decaying away from the diffraction grating. Thus, in the dielectric region D_1,

$$\gamma^+ = \gamma_d^+ \qquad \text{and} \qquad \gamma^- = \gamma_d^- \tag{36}$$

in the dielectric region D_2,

$$\gamma^+ = -\gamma_d^+ \qquad \text{and} \qquad \gamma^- = -\gamma_d^- \tag{37}$$

in the vacuum region V,

$$\gamma^+ = \gamma_v^+ \qquad \text{and} \qquad \gamma^- = \gamma_v^- \tag{38}$$

and in the plasma region P,

$$\gamma^+ = \gamma_p^+ \qquad \text{and} \qquad \gamma^- = \gamma_p^- \tag{39}$$

where

$$\gamma_v^+ = \sqrt{(k_y+q)^2 - \frac{\omega^2}{c^2}} \quad \text{and} \quad \gamma_v^- = \sqrt{(k_y-q)^2 - \frac{\omega^2}{c^2}} \tag{40}$$

$$\gamma_d^+ = \sqrt{(k_y+q)^2 - \frac{\omega^2}{c^2}\varepsilon_d} \quad \text{and} \quad \gamma_d^- = \sqrt{(k_y-q)^2 - \frac{\omega^2}{c^2}\varepsilon_d} \tag{41}$$

$$\gamma_p^+ = \sqrt{(k_y+q)^2 - \frac{\omega^2}{c^2}\varepsilon_p} \quad \text{and} \quad \gamma_p^- = \sqrt{(k_y-q)^2 - \frac{\omega^2}{c^2}\varepsilon_p} \tag{42}$$

Note that for a normal incident wave, $\theta = 0$, and $H^+ = H^-$, which simplifies the computations.

General conditions for resonant signal transmission via a multy-layer structure were studied in [6], [7], [8]. In those papers, the transmission problem was formulated by the method of transfer functions, where each uniform region is represented by a 2×2 matrix. This method leads to a cumbersome and computationally intense result, which is represented in the form of a product of all of these matrices and their inverses. Instead, we used the impedance method which relates the refection coefficient to the load impedance of the vacuum region located to the left of the plasma. In particular, if that load impedance equals the vacuum impedance then the refection coefficient vanishes and total transmission occurs.

The impedance method has been used in the literature to study propagating waves. We have adapted this method to include evanescent waves. The main advantage of the impedance method is in its simplicity. It uses a recursion formula to find the impedance at each interface, which lends itself to generalizations to any number of layers and to non-homogeneous plasmas. We have first applied this method to two- and three-layer structures. It enabled us to obtain analytical and numerical results for electromagnetic wave transmission in a simple and transparent way and gain a better understanding of that process.

Our results concerning multi-layer structures and a detailed discussion of the impedance method can be found in the following publications:

(i) N. Sternberg and A. I. Smolyakov, Resonant transmission of electromagnetic waves in multilayer dense plasma structures, IEEE Trans. Plasma Sci., 37, 7, 2009.

(ii) N. Sternberg and A. Smolyakov, Resonant transparency of a three-layer structure containing the dense plasma region, Progress In Electromagnetics Research, PIER 99, 37–52, 2009.

In an attempt to understand the effects of the diffraction grating, we have applied the impedance method to the model presented above and obtained analytical transparency conditions which show that the total wave transmission can indeed be achieved through surface wave resonance with the sidebands for an appropriate choice of the parameters of the diffraction grating. Those theoretical results are being prepared for publication.

3. Effects of the incidence angle and diffraction grating parameters on resonant wave transmission through dense plasma without dissipation

Studying the diffraction grating model described in the previous section, we have found that resonant transparency can be induced if the appropriate parameters are controlled. There are seven parameters of the problem: the dielectric permittivity ϵ_d, the diffraction grating permittivity ϵ_g, the position of the diffraction grating $z = a$, the diffraction grating wave vector q, the diffraction grating modulation amplitude α, the diffraction grating width h_g and the incidence angle θ. Our analysis has shown that the parameter a is a function of the plasma parameters, the incidence angle and the dielectric permittivity. In our computations, we set $h_g = 0.05$m, and we assumed a metal diffraction grating, i.e., $\epsilon_g > 0$. In practice, ϵ_g is fixed and $\alpha < \epsilon_g$. For a plasma without dissipation, we chose $\epsilon_g = 2$ and $\alpha = 1.8$.

We were able to demonstrate that resonant transparency can be achieved by adjusting q and placing the diffraction grating appropriately. This is illustrated in Fig. 4 for $\epsilon_d = 1.2$ and different incidence angles. The resonant position of the diffraction grating depends on the incidence angle. In fact, a larger incidence angle requires a larger resonant value of a. For example, $a = 0.06$m for $\theta = 0$, $a = 0.07$m for $\theta = \pi/6$, $a = 0.08$m for $\theta = \pi/4$, and $a = 0.09$m for $\theta = \pi/3$.

Fig. 5 shows that the position of the diffraction grating plays a significant role in resonant signal transmission. Perturbations in the parameter a can significantly reduce signal transmission. In practice, however, the exact incidence angle is not known, and therefore the effects of the incidence angle need to be reduced. Fig. 6 shows that this can be done by increasing the dielectric permittivity. For larger dielectric permittivity the effect of the distance from plasma to diffraction grating for different incidence angles is less pronounced. In addition, larger values of ϵ_d require smaller values of a for achieving resonance, which may be important in application. Fig. 7 shows that adequate signal transmission can indeed be achieved for all incidence

angles by choosing an average value of a and a sufficiently large ϵ_d.

More detailed theoretical results are being prepared for publication.

4. The diffraction grating model with plasma dissipation.

All our results so far neglected the electron-atom collisions in the plasma, i.e., $\nu_e = 0$. In practice, however, electron-atom collisions cannot be neglected. Figs. 8 and 9 show the reflection, transmission and absorption coefficients for $\theta = 0$ and $\theta = \pi/3$ if there is no diffraction grating present. As can be seen, for small values of ν_e, most of the signal is reflected by the plasma, some is absorbed, and very little is transmitted. Indeed, for $\nu_e = 0.01$ and $\theta = 0$, the reflection coefficient is $R = 99.4\%$, while the transmission coefficient is about $T = 0.3\%$, and the absorption coefficient is about $A = 0.3\%$. This behavior is similar as the angle is increased. For example, for $\nu_e = 0.01\omega$ and $\theta = \pi/3$, we have found $R = 98.4\%$, $T = 0.8\%$, and $A = 0.8\%$. For intermediate values of ν_e most of the signal is absorbed by the plasma. Eventually, for large values of ν_e much of the signal is transmitted. This is due to the fact, that when plasma dissipation is taken into account, ϵ_p becomes complex, and as shown in Fig. 10 the plasma eventually becomes a dielectric for sufficiently large values of ν_e.

Using a diffraction grating can significantly improve signal transmission, although resonant transmission cannot be achieved. For our computations we chose $\epsilon_d = 1.2$, $\epsilon_g = 4$, $\alpha = 3.8$ and $a = 0.062$m. Figs. 11–13 show the transmission, reflection and absorption coefficients, respectively, for $\nu_e = 0.01\omega$. In this case, the corresponding extreme values are $T_{max} = 43\%$, $R_{min} = 14\%$ and $A_{max} = 43\%$. In comparison, for $\theta = \pi/3$, $T_{max} = 50\%$, $R_{min} = 6\%$ and $A_{max} = 44\%$. Thus, by increasing the incidence angle, the reflection coefficient can be decreased and the transmission coefficient can be increased. Observe that the bandwidth is wider when dissipation is present, but by increasing the incidence angle, the bandwidth becomes even wider (see Fig. 14).

It is interesting to note, for both, $\theta = 0$ and $\theta = \pi/3$ there is only one value of the diffraction grating wave vector q which will produce partial signal transmission. We have found that this is the case for $\theta > 5\pi/21$. For $\theta < 5\pi/21$ two such values of q exist. The transition from one resonant value of q to two such values takes place at about $\theta = 5\pi/21$. As the incidence angle is decreased, the two values of q are moving closer to each other, and eventually coalesce for $\theta = 0$. This is illustrated in Figs. 15–17.

Our results on dissipation are preliminary. More studies are needed in this direction.

5. Transmission of electromagnetic waves through dense plasma using a barrier.

In order to model transmission of electromagnetic waves using a barrier instead of a diffraction grating, we set $\alpha = 0$ in equation (12). Deriving analytical transparency condition, we were able to show that using this simple model one can obtain resonant transmission. The resonance in this case is due to standing waves rather than to surface modes. We have studied this type of resonance in [9]. Accounting for plasma dissipation, we were able to demonstrate reflectionless transmission and absorbtion of the incident electromagnetic wave.

Our results are being prepared for publication.

6. Transmission of electromagnetic waves through periodic plasma modulation.

The idea of using a periodic modulation to transmit electromagnetic waves trough opaque material can be found in [10]. We were able to demonstrate that this method can be adapted to transmit a signal of GPS frequency through dense plasma. While in [10], the whole plasma layer was modulated, we have found that transparency can also be achieved by partial plasma modulation.

In order to model signal transmission through dense plasma by periodic modulation, we simplified equation (12) by setting $\epsilon_g = 0$. We are currently in the process of developing a theoretical framework which will clarify what types of resonance are possible in this situation. Our goal is to describe the relationship between the modulation amplitude and the modulation wave vector which are needed to induce resonance. We intend to submitted our results for publication.

7. Modeling two-dimensional bounded plasmas for intermediate and high pressures.

In order to successfully address the communication black-out problem during hypersonic fight, one needs to understand the plasma that forms around the vehicle in this situation. We started by modeling a bounded collsionless cylindrical plasma and studied the behavior of the plasma parameters. We are currently modeling a bounded two-dimensional cylindrical plasma which takes ion-atom and electron-atom collisions into account [11]. Our preliminary results were presented at the 2009 Workshop on Radio-Frequency Discharges, La Presqu'il de Giens, France, 17–20 May, 2009. A

paper on two-dimensional collisional gas discharge plasma is being prepared for publication in collaboration with V. Godyak.

8. Plasma under the action of a magnetic field.

It is possible that a plasma modulation can indeed be obtained using a weak magnetic field. In general, it is suggested in the literature that a magnetic field can facilitate signal transmission. For practical applications, this magnetic field has to be weak. The question arises: What does "weak" really mean?

In [12], we have studied a cylindrical plasma with an axial magnetic field, and found that sufficiently weak magnetic fields have no effect on plasmas and in this case the Boltzmann equilibrium for the electrons can be assumed. For our computations, we used a Ramsauer gas. In [13], we extended this result to non-Ramsauer gases and found simple transformation formulas between the corresponding results. We have found that in both cases, for Ramsauer and non-Ramsauer gases, if the plasma is affected by a strong magnetic field, then the Boltzmann equilibrium is violated. We introduced a magnetic parameter that gives a criterion for the cut-off between the two regions of weak (practically negligible) and strong (dominant) magnetic field effects.

We are currently conducting similar investigations in order to understand the effects of an oblique magnetic field on bounded plasmas and the transition from weak to strong magnetic fields. This work is done in collaboration with J.-L. Raimbault, Ecole Polytechnique, Paris, France and will be published.

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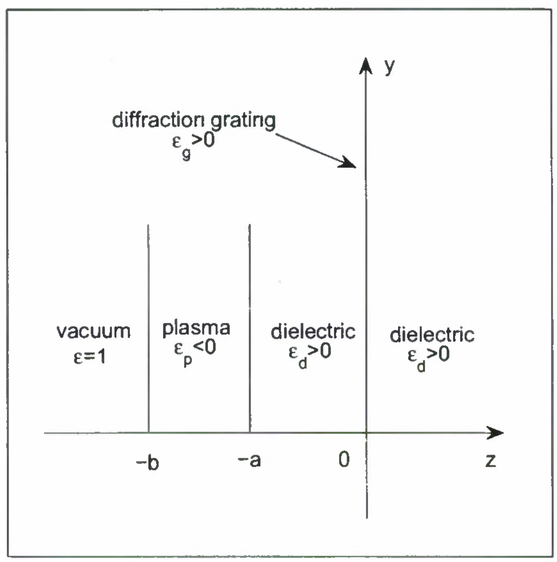

Figure 1: Schematic representation of the multi-layer problem with a diffraction grating.

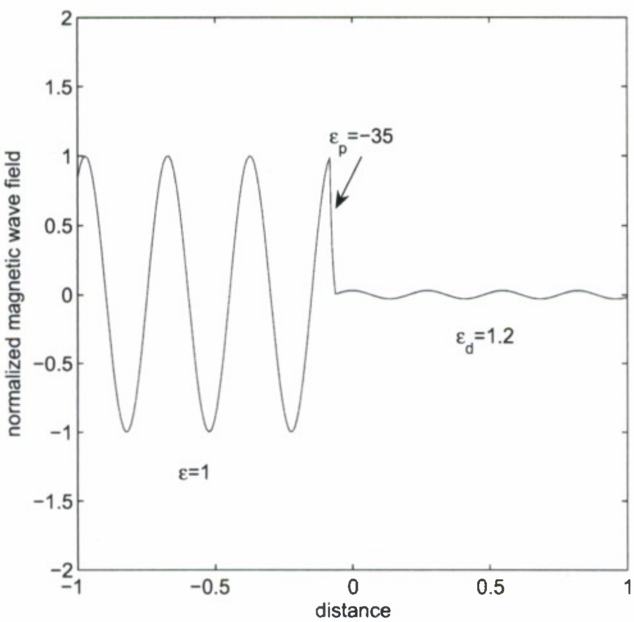

Figure 2: A propagating normal incidence wave in the vacuum layer enters the dense plasma layer becoming evanescent (red) and continues propagating in the dielectric layer with a small amplitude.

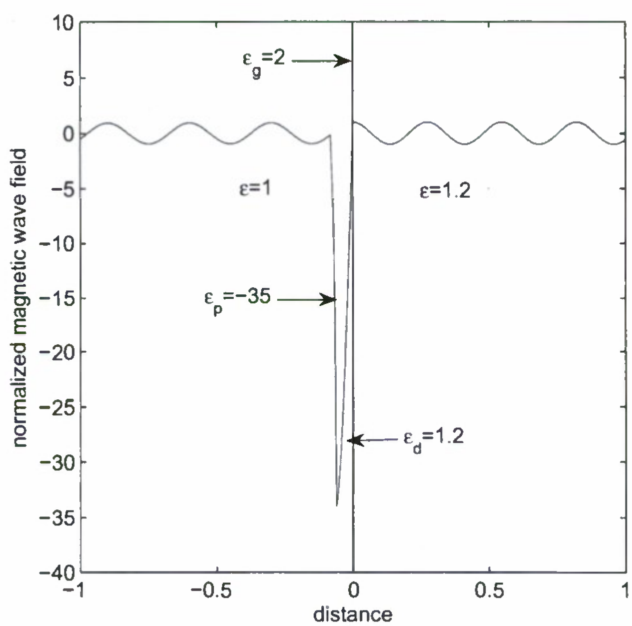

Figure 3: Resonant electromagnetic wave transmission through a dense plasma layer using a diffraction grating for normal incidence.

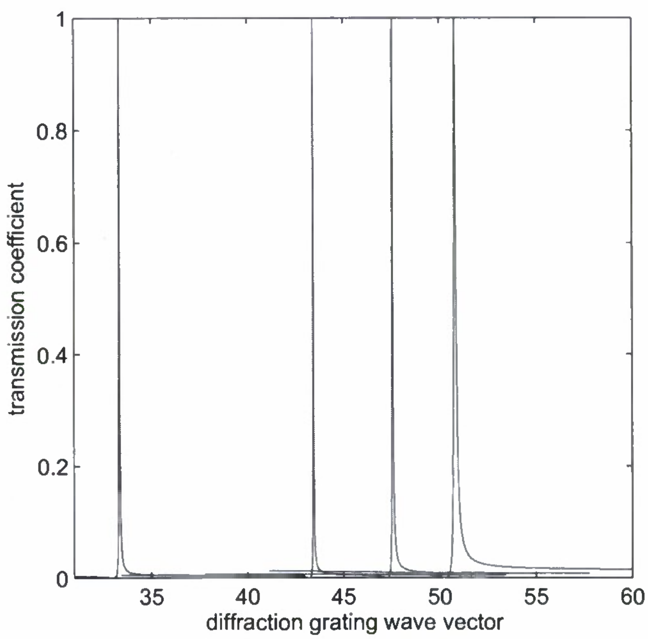

Figure 4: Transmission coefficient versus diffraction grating wave vector for $\epsilon_d = 1.2$ and for different incidence angles: $\theta = 0$ (blue), $\theta = \pi/6$ (red), $\theta = \pi/4$ (green), and $\theta = \pi/3$ (purple).

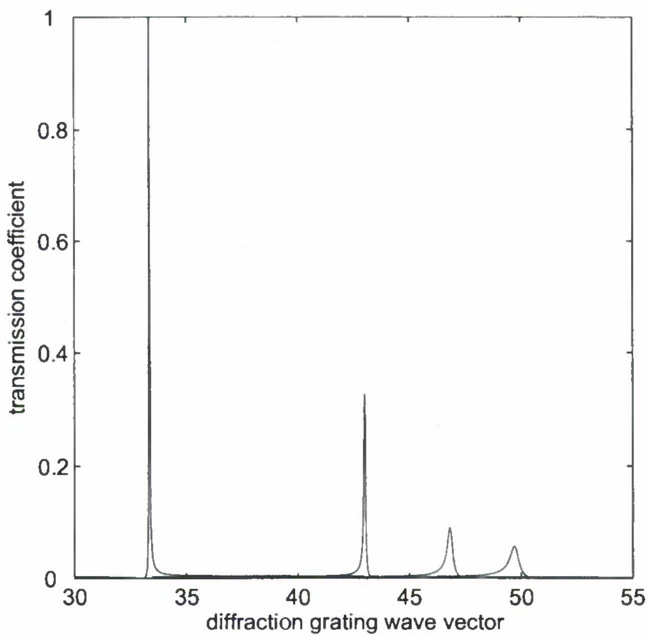

Figure 5: Transmission coefficient versus diffraction grating wave vector for $\epsilon_d = 1.2$, $a = 0.06$m, and for different incidence angles: $\theta = 0$ (blue), $\theta = \pi/6$ (red), $\theta = \pi/4$ (green), and $\theta = \pi/3$ (purple).

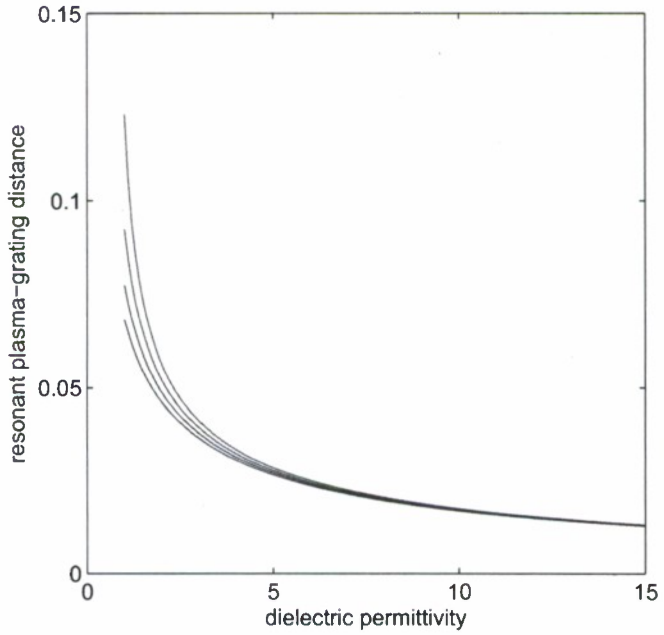

Figure 6: Resonant distance from the plasma layer to the diffraction grating versus dielectric permittivity for different incidence angles: $\theta = 0$ (blue), $\theta = \pi/6$ (red), $\theta = \pi/4$ (green), and $\theta = \pi/3$ (purple).

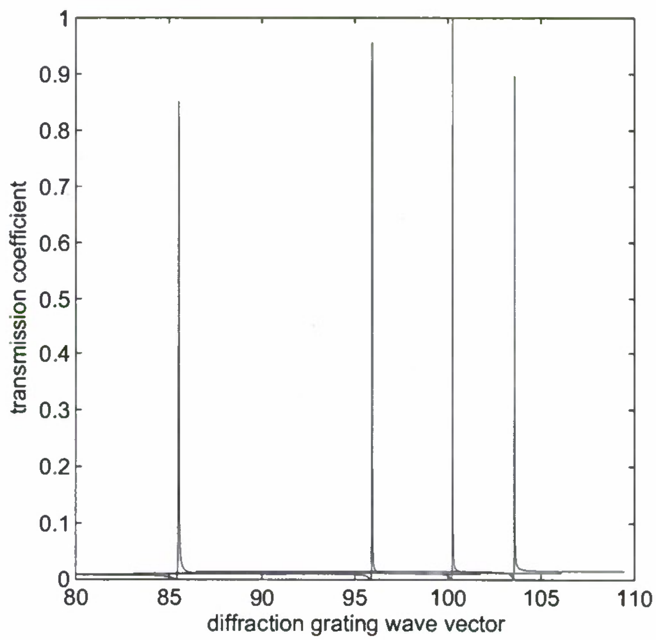

Figure 7: Transmission coefficient versus diffraction grating wave vector for $\epsilon_d = 10$, $a = 0.0172$m, and for different incidence angles: $\theta = 0$ (blue), $\theta = \pi/6$ (red), $\theta = \pi/4$ (green), and $\theta = \pi/3$ (purple).

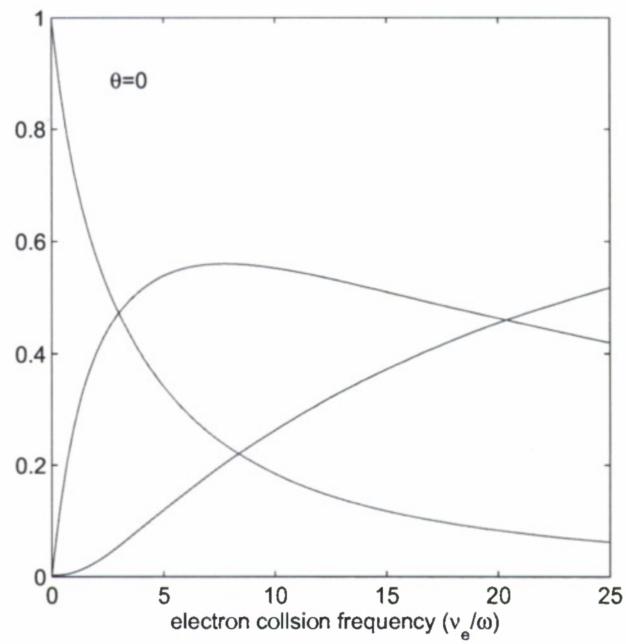

Figure 8: Transmission coefficient (blue), reflection coefficient (green) and absorption coefficient (red) versus normalized electron collision frequency for $\theta = 0$ when no diffraction grating is present.

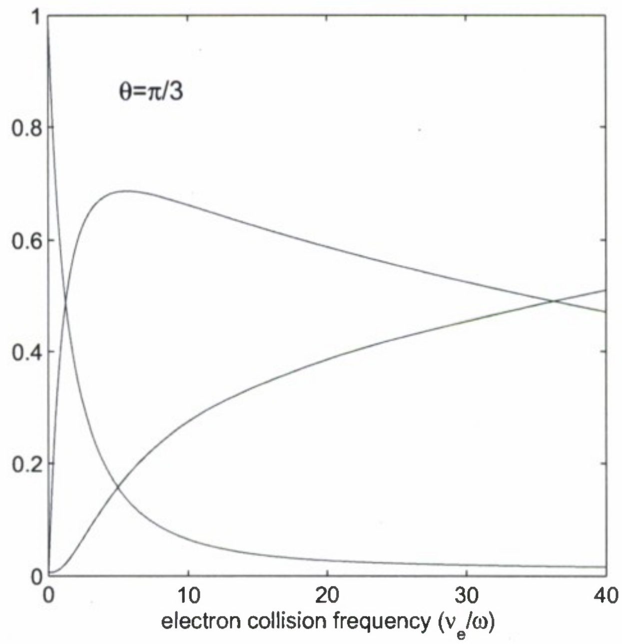

Figure 9: Transmission coefficient (blue), reflection coefficient (green) and absorption coefficient (red) versus normalized electron collision frequency for $\theta = \pi/3$ when no diffraction grating is present.

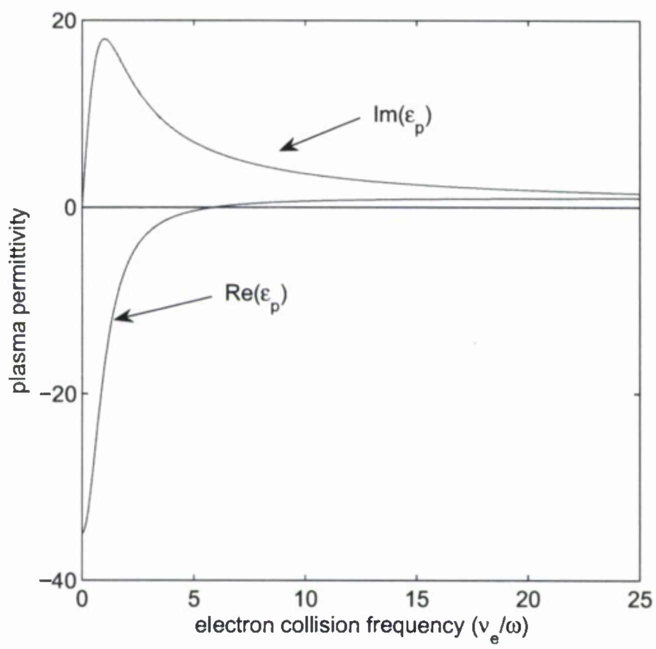

Figure 10: Real and imaginary part of the plasma permittivity versus the normalized electron collision frequency.

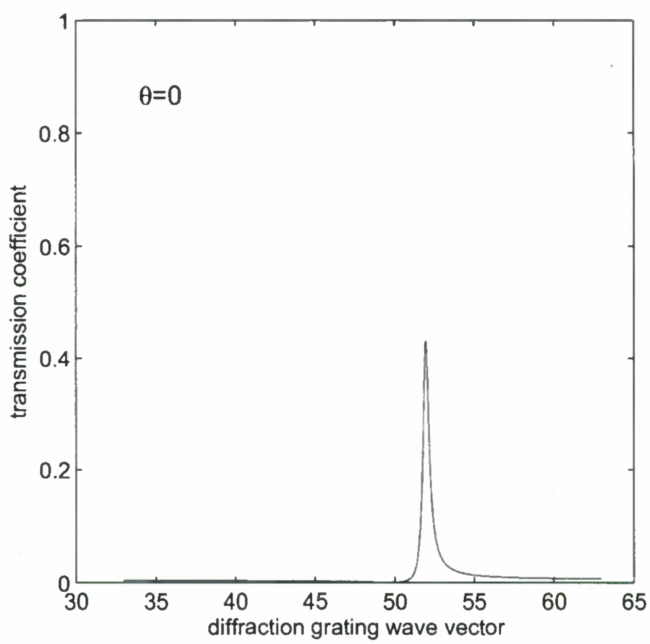

Figure 11: Transmission coefficient versus the diffraction grating wave vector for $\nu_e = 0.01\omega$ and $\theta = 0$.

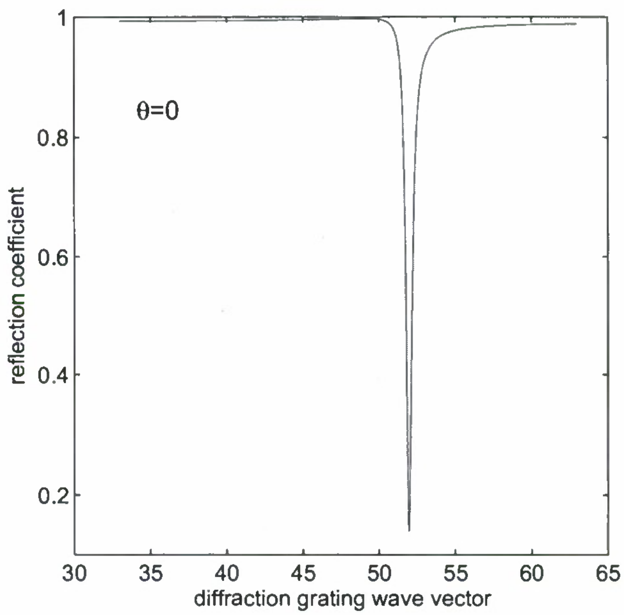

Figure 12: Reflection coefficient versus the diffraction grating wave vector for $\nu_e = 0.01\omega$ and $\theta = 0$.

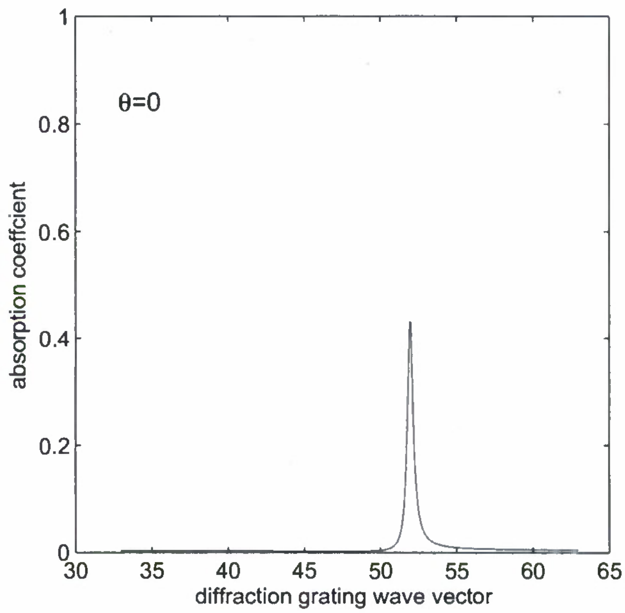

Figure 13: Absorption coefficient versus the diffraction grating wave vector for $\nu_e = 0.01\omega$ and $\theta = 0$.

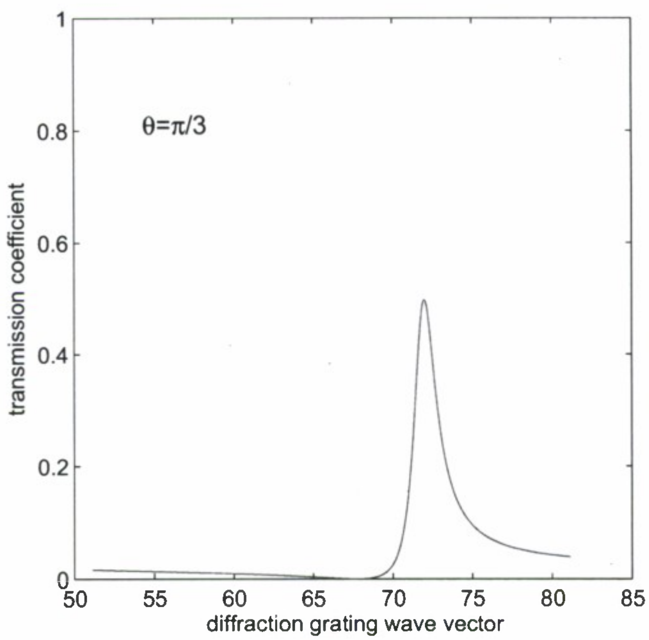

Figure 14: Transmission coefficient versus the diffraction grating wave vector for $\nu_e = 0.01\omega$ and $\theta = \pi/3$.

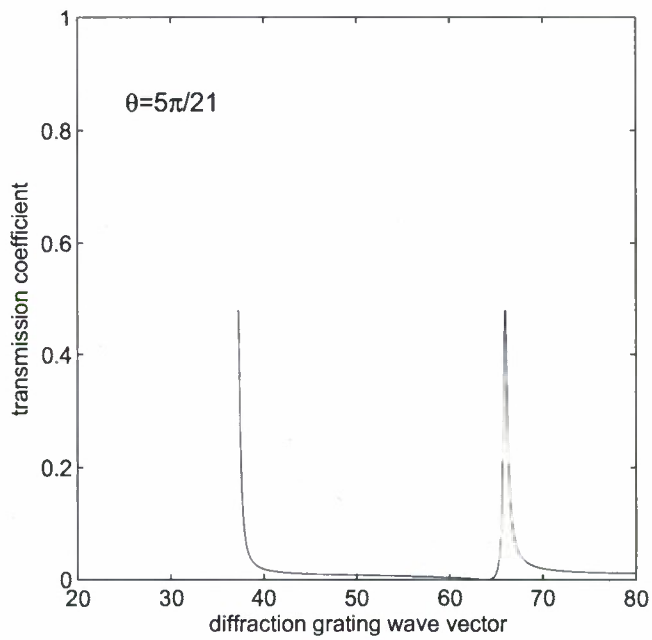

Figure 15: Transmission coefficient versus the diffraction grating wave vector for $\nu_e = 0.01\omega$ and $\theta = 5\pi/21$.

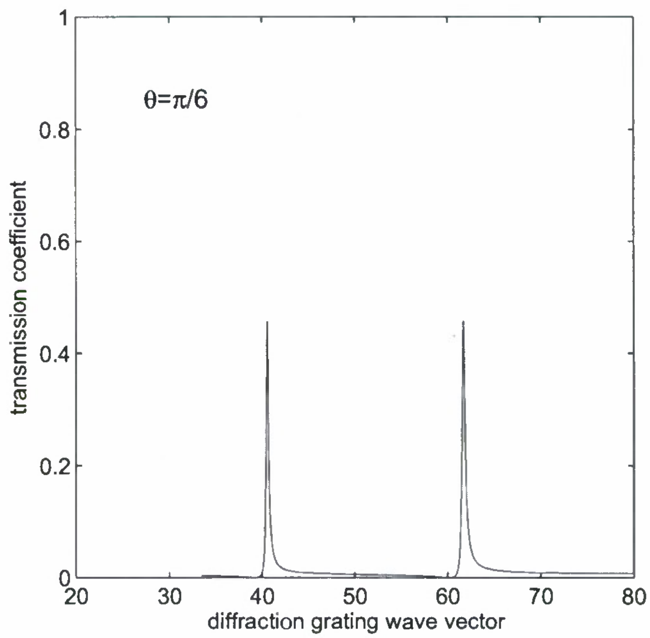

Figure 16: Transmission coefficient versus the diffraction grating wave vector for $\nu_e = 0.01\omega$ and $\theta = \pi/6$.

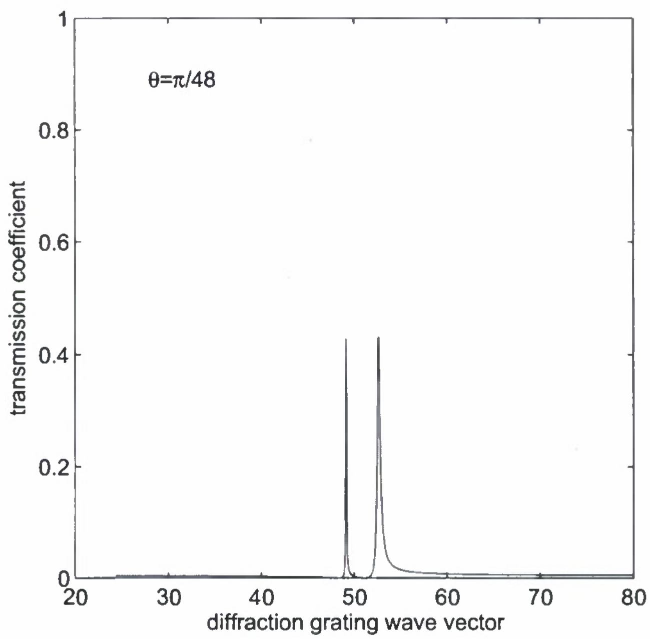

Figure 17: Transmission coefficient versus the diffraction grating wave vector for $\nu_e = 0.01\omega$ and $\theta = \pi/48$.